$a_n = a_1 \cdot r^{n-1}$

43. 9, 18, 36, 72, 144

$144 = 9 \cdot r^4$

$16 = r^4$

$\pm 2 = r$

37.

$a_1 = 2000$

$a_2 = 1600$

$r = \frac{4}{5}$

$a_6 = 2000\left(\frac{4}{5}\right)^5$

655.36 lb

38.

$a_1 = 40{,}000$

$a_5 = $ ____

$r = 1.04$

$a_5 = 40{,}000(1.04)^4$

46,794.34

11-4 Geometric Series

sum of a geometric sequence

$2 + 6 + 18 + 54 + 162$

$r = 3$

$S_5 = 2 + 6 + 18 + 54 + 162$

$-3 \cdot S_5 \quad -6 - 18 - 54 - 162 - 486$

$-2 S_5 = 2 - 486$

$S_5 = \frac{2 - 486}{-2}$

$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$

$a_n = a_1 r^{n-1}$

$\frac{a_1 - r \cdot a_1 r^{n-1}}{1 - r}$

$\frac{a_1 - a_1 r^n}{1 - r}$

$$S_n = \frac{a_1 - r\,a_n}{1 - r}$$

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} \qquad r \neq 1$$

same

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Ex 1:
Find the sum.
$a_1 = 7776$
$a_n = 6$

$r = \frac{-1}{6}$

$$S_n = \frac{7776 - \left(\frac{-1}{6}\right)(6)}{\left[1 - \frac{-1}{6}\right]}$$

6666

Ex 2:

$a_1 = ?$
$S_8 = 765$

$r = 2$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$765 = \frac{a_1(1 - 2^8)}{1 - 2}$$

$3 = a_1$

Sigma Notation

$$\sum_{n=1}^{12} 3 \cdot 2^{n-1}$$

Expanded Form

$3 + 6 + 12 + 24 + \ldots\ldots$ $+6144$

$$\text{Sum} = \frac{3 - (2)(6144)}{1 - 2}$$

$S_{12} = 12285$

Sigma Notation

$$\sum_{n=1}^{5} 6 \cdot \left(\tfrac{1}{3}\right)^{n-1}$$

$= 6 + 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

Put the following series into sigma notation.

$48 + 24 + 12 + 6 + \ldots + \frac{3}{8}$

$a_n = a_1 \cdot r^{n-1}$

$a_n = 48 \cdot \frac{1}{2}^{n-1}$

$\frac{3}{8} = 48 \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{128} = \left(\frac{1}{2}\right)^{n-1}$

$7 = n - 1$

$8 = n$

$$\sum_{n=1}^{8} 48 \cdot \left(\tfrac{1}{2}\right)^{n-1}$$

29. **HEALTH** Contagious diseases can spread very quickly. Suppose five people are ill during the first week of an epidemic and that each person who is ill spreads the disease to four people by the end of the next week. By the end of the tenth week of the epidemic, how many people have been affected by the illness?

$a_1 = 5$

$r = 4$

$S_{10} = \frac{5(1-4^{10})}{1-4}$

$= 1,747,625$

HW
p597
15-27odd, 30, 33, 35, 39, 41, 43, 47